Towards the design of a feasible seismic metabarrier using multi-mass resonators

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Metamaterials: Periodic and locally resonant media

Periodic Media

Locally resonant materials

Wave filtering at wavelength
\[ \lambda \approx a \quad \text{a periodicity length} \]

Wave filtering at subwavelength scale
\[ f \approx f_r \quad \text{resonance frequency} \]
Periodic and locally resonant media:

Multi-scale applications


Brule, PRL (2014)
Seismic excitation & Building Natural Frequencies

Northridge earthquake spectrum (1994)

1 – 10 Hz  \( \Rightarrow \lambda = 10 – 500 \text{ m} \)

Wave’s control at subwavelength scale
Seismic Rayleigh Waves

http://www.bgs.ac.uk/discoveringGeology/hazards/earthquakes

http://web.ics.purdue.edu/~braile/edumod/waves/Rwave.htm

Displacement seismograph

http://www.bgs.ac.uk/discoveringGeology/hazards/earthquakes
Rayleigh waves and local resonances: Surface waves deflection by forest trees

The idea: a Seismic MetabARRIER for Rayleigh waves

Seismic Metabarrier: Resonator Design
Modeling approach

• Analytical Approach

• Numerical Model

• Experimental Verification
Analytical model: 3-Mode Resonators

1. horizontal  
2. vertical  
3. rotational

Rayleigh waves
Pressure and shear potentials:

\[ \phi = Ae \sqrt{\frac{kz \left(1 - \frac{\omega^2}{k^2c_L^2}\right) + i(\omega t - kz)}{k}} \]
\[ \psi = Be \sqrt{\frac{kz \left(1 - \frac{\omega^2}{k^2c_S^2}\right) + i(\omega t - kz)}{k}} \]

Wave equation:

\[ \nabla^2 \phi = \frac{1}{c_L^2} \frac{\partial^2 \phi}{\partial t^2} \]
\[ \nabla^2 \psi = \frac{1}{c_S^2} \frac{\partial^2 \psi}{\partial t^2} \]

Boundary conditions:

\[ \sigma_{zz}(0) = \sigma_{zz}^{(2)} \]
\[ \sigma_{xz}(0) = \sigma_{xz}^{(1)} + \sigma_{xz}^{(3)} \]
Analytical model: Dispersion Relation

Bulk shear waves

$c_R$
$c_L$
$c_S$

Band Gap

$c \geq c_S$

Bulk shear wave

$c_S$
Upper and lower edge Band Gap Limits:

Vertical mode:

$$ f^-: \ k \rightarrow \infty \quad f^- = f_v $$

$$ f^+: \ k = \frac{\omega}{c_s} \quad f^+ = f_v (\beta + \sqrt{\beta^2 + 1}) $$

Non-Dimensional parameter:

$$ \beta = \frac{m \omega_v}{2 A \rho_s c_{S,soil}} \sqrt{1 - \frac{c_{S,soil}^2}{c_{L,soil}^2}} $$

$$ \Delta \Omega = \frac{2 \left( \beta + \sqrt{\beta^2 + 1} - 1 \right)}{\left( \beta + \sqrt{\beta^2 + 1} + 1 \right)} $$
Analytical model: FE validation

Seismic Metabarrier

Harmonic Excitation [1 Hz]
Analytical model: FE validation

Seismic MetabARRIER

Harmonic Excitation [3 Hz]
Case Study: Analytical prediction

Resonator

\( f_v = 4.9 \text{ Hz} \)
\( m = 6.6 \text{ ton} \)

Soil

\( c_{L,\text{soil}} = 230 \frac{m}{s} \)
\( c_{S,\text{soil}} = 120 \frac{m}{s} \)

\( \beta = 0.37 \)
Case Study: FEM simulation

Resonators arrangement

Harmonic source $u(t), w(t)$

[1-10] Hz

Source [5.0 Hz]

Output

Resonators

PML

12 Resonators
Attenuation 60%

Transmission @5Hz [a.u.]

Number of Resonators Line
12 Resonators

$\beta_{exp} = 0.37$

$BG = 11.0 - 16 \text{ kHz}$
Seismographs: free propagation vs. resonators

- **Free Propagation**
  - Measurements line
  - Wave front
  - Boundary reflections
  - Initial pulse $c_r$

- **Resonators**
  - Boundary reflections
  - Resonators
  - Initial pulse
Frequency analysis

Initial pulse

Transmitted pulse

Transmitted frequency content

Res.

no Res.

11.0-16 kHz

BG

0.60
Performance of the metabARRIER

\[ \uparrow \frac{m}{A} \uparrow BG \uparrow c_s \downarrow BG \]
Multi-mass metabARRIER for broadband attenuation

- Multi-mass resonators for multi-frequency attenuation

Rayleigh waves
Pressure and shear potentials:
\[
\phi = Ae \sqrt{\frac{kz}{k^2 c_L^2}} \left(1 - \frac{\omega^2}{k^2 c_L^2} + i(\omega t - kz)\right)
\]
\[
\psi = Be \sqrt{\frac{kz}{k^2 c_S^2}} \left(1 - \frac{\omega^2}{k^2 c_S^2} + i(\omega t - kz)\right)
\]

Boundary conditions:
\[
\sigma_{zz} = \sigma_{zz, \text{res}} \quad \text{Modal approach}
\]
\[
\sigma_{xz} = 0
\]
Multi-mass metabARRIER for broad band attenuation

- Multi-mass resonators for multi-frequency attenuation

\[ m_2 = 3.5 \text{ ton} \]

\[ m_1 = 7 \text{ ton} \]
Multi-mass metabarrier for broad band attenuation: Performance

Identical total mass with half length
Rainbow trapping and metawedge

Rainbow trapping


Metawedge

A Multi-mass metawedge

Single-Mass Metawedge

\[ f_{r,0}, f_{r,n} \]

Double-Mass Metawedge

\[ f_{r,m}, f_{r,n} \]

2\textsuperscript{nd} Mode

\[ f_{r,m}, f_{r,n} \]

1\textsuperscript{st} Mode

\[ f_{r,0} = 2.4 \text{ Hz} \]

\[ f_{r,n} = 13.4 \text{ Hz} \]
Optimal multi-mass metabarrier for broad band attenuation

\[ f_{r,1}, \ldots, f_{r,n} \]

\[ \Delta f_{r,1}, \ldots, \Delta f_{r,n} \]

Input

Optimization

Output

\[ Min \left( \sum_m \right) \]

Structural analysis

\[ f_{r,1}, \ldots, f_{r,n} \]

\[ \Delta f_{r,1}, \ldots, \Delta f_{r,n} \]
Optimal multi-mass metabarrier for broad band attenuation

\[
\begin{align*}
    m_4 &= 3.0 \text{ ton} \\
    m_3 &= 4.0 \text{ ton} \\
    m_2 &= 5.6 \text{ ton} \\
    m_1 &= 4.7 \text{ ton}
\end{align*}
\]

\[
\sum m/A < 20 \text{ ton/m}^2
\]

\[
\begin{align*}
    f_{r,1} &= 2.27 \text{ Hz} \\
    f_{r,2} &= 4.48 \text{ Hz} \\
    f_{r,3} &= 7.33 \text{ Hz} \\
    f_{r,4} &= 12.99 \text{ Hz}
\end{align*}
\]
Main results

Main results
• Design of seismic metabarrier for surface wave attenuation
• “Proof of concept” validation on a scaled down model
• Analytical and numerical studies show achievable large attenuation (60%) in the low frequency range Hz with compact single and multi-mass metabarriers

Limitations
• «Ideal soil»: - linear elastic; -isotropic/homogenous; - flat/regular surface
  The complex propagation pattern of seismic waves in real soils and the full interaction between soil and metamaterial cannot be captured.
• Elastic resonator: energy dissipation and material nonlinearities can influence the dynamic response of the resonator.
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